RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. THIRD SEMESTER EXAMINATION, DECEMBER 2024

SECOND YEAR [BATCH 2023-27]

MATHEMATICS

Date : 06/12/2024

Time : 11 am – 1 pm

PAPER : 3MTMMIC1

Full Marks : 50

[Use a separate Answer Book for **each Group**]

Group A

$\mathbb{N}, \mathbb{Q}$ and $\mathbb{R}$ denote the set of all natural numbers, rationals and real numbers respectively. All other symbols bear their usual meanings.

Answer **any five** questions: [5×5]

1. Let $A=\{x\in\mathbb{R}: 0\le x\le 10\}$, $B=\{x\in\mathbb{R}: 5\le x\le 15\}, C=\{x\in\mathbb{R}: 5\le x\}$

   a) Verify that $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$.

   b) Find $A\bigcap B\bigcap C^c$. [C^c is the complement of C] [3+2]

2. Let S be a set of all positive divisors of 30. Define a relation ρ on S as $a\rho b$ if a divides b. Show that ρ is a partial order relation on S. [5]

3. Let f & g be two functions from $\mathbb{R}$ to $\mathbb{R}$ defined by

   $f(x)=|x|+x \ \ \forall x\in\mathbb{R}$ and $g(x)=|x|-x \ \ \forall x\in\mathbb{R}$

   Find fog and gof. [5]

4. State Archimedean property of real numbers. Use it to show that there is an integer z such that $z\le x<z+1$ for any $x\in\mathbb{R}$. [1+4]

5. a) State least upper bound axiom of $\mathbb{R}$.

   b) Find the supremum and infimum of the set $A=\{x\in\mathbb{R}: 3x^2-9<0\}$. [1+4]

6. Use the principle of mathematical induction to prove that $3^{4n+2}+5^{2n+1}$ is divisible by 14 for all $n\in\mathbb{N}$. [5]

7. Prove that every infinite bounded subset of $\mathbb{R}$ has at least one limit point in $\mathbb{R}$. [5]

8. Find the interior and the set of all limit points of the set

   $A=(-1,0)\cup\left\{\frac{1}{m}+\frac{1}{n}: m,n\in\mathbb{N}\right\}$ [2+3]

Group B

Answer **any five** questions: [5×5]

9. If z be a complex number such that $\frac{z+1}{z-i}$ is purely imaginary, show that z lies on the circle having $-\frac{1}{2}+\frac{1}{2}i$ as centre. What is its radius? [4+1]

10. Find the product of all the values of $(1+i)^{\frac{4}{5}}$. [5]

11. Show that the principal value of $\left(\frac{i}{-i}\right)^{i}$ is equal to the ratio of the principal values of i^{i} and $(-i)^{i}$. [5]

12. a) State Fundamental Theorem of Algebra.

    b) Prove that the equation

    $\frac{1}{x+a_1}+\frac{1}{x+a_2}+....+\frac{1}{x+a_n}=\frac{1}{x}$ where $a_1,a_2,a_3,......\ a_n$ are all negative real numbers, cannot have an imaginary root. [1+4]

13. Find the condition that the roots of the equation $ax^3 + 3bx^2 + 3cx + d = 0$ will be in A. P. [5]

14. If α,β,γ are the roots of the equation $x^3+px^2+qx+r=0\left(r\neq 0\right)$ find the equation whose roots are $\alpha\beta+\frac{1}{\gamma},\beta\gamma+\frac{1}{\alpha},\gamma a+\frac{1}{\beta}$. [5]

15. Solve by Cardan's method : $x^3 - 18x - 35 = 0$. [5]

16. Prove that $1!\ 3!\ 5!\ .....(2n-1)! > (n!)^n$, where n is a positive integer. [5]

Or

If n is a positive integer greater than 1, prove that $2^n > 1 + n.2^{\frac{(n-1)}{2}}$ [5]

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